

Skript zum Versuch

A40

Viscosity of Liquids: Höppler's and Ostwald's Viscometers

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1 The Task

Using a falling sphere viscometer (Höppler-viscometer), the viscosity of water at 25, 30, 40 and 50 °C has to be determined. With a capillary viscometer (Ostwald-viscometer) the viscosity of water, ethanol and a mixture of water and ethanol at 20 °C has to be measured.

2 Introduction

2.1 General Principles

As a medium flows parallel to a rigid wall – like e.g. the flow in a tube – then the molecules near the wall move more slowly in flow direction than those farther apart from the wall. The reason for this are the attractive interactions between molecules and the wall: the first molecular layer „sticks“ to the wall, whereas the more distant layers move forward under the action of a pressure gradient along the tube. This situation is sketched in fig. 1.

Let z be the direction of the flow and x the coordinate normal to it then the flow velocity component in z -direction exhibits a gradient in x -direction:

$$\frac{dv_z}{dx} \neq 0 \quad (1)$$

This velocity gradient is the driving force for the momentum exchange or to put it in another way the momentum transport between the layers: the z -component of the

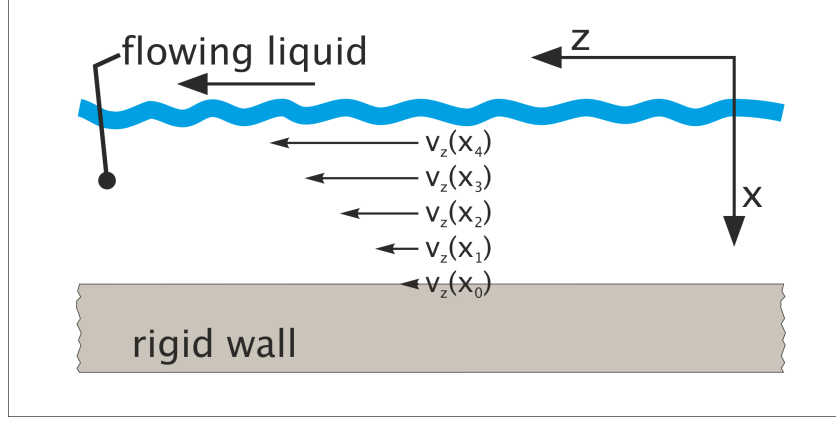


Fig. 1: Schematic representation of a moving fluid in contact with a rigid wall. The z -component of the flow velocity gets bigger the further the layer is off the wall.

momentum vector \vec{p} will be transported in x -direction. The proportionality factor is the *dynamic viscosity* η of the fluid.

For a microscopic interpretation of the phenomenon, one looks at the molecules within layer no. n that moves in z -direction. If a molecule jumps to an adjacent layer (for example, by diffusion in x -direction), then it carries its momentum $p_z(n)$ from the n^{th} into the layer $n + 1$. There it decelerates (or accelerates) that adjacent layer. The amount of momentum transported per unit of time, which can be regarded as friction, is proportional to the velocity gradient and area cross-section A . The respective transport law can be formulated in an analogous way like Fick's 1. law of diffusion:

$$\frac{dp_z}{dt} = F_R = -\eta A \frac{dv_z}{dx} \quad (2)$$

The z -component of the momentum will be transported in x -direction. If the viscosity is independent of the velocity then the fluid is denoted as *Newtonian fluid*.

2.2 H ppler's falling sphere viscometer

H ppler's¹ falling sphere (or ball) viscometer utilizes Stoke's equation to determine the dynamic viscosity η . For the frictional force F_R for a sphere in a laminar flow one has:

$$F_R = 6\pi r v \eta \quad (3)$$

This friction force is in equilibrium with the gravitational weight of the sphere, F_G , reduced by the buoyant force, F_A , in the liquid:

$$\begin{aligned} F_R &= F_G - F_A \\ F_R &= (\rho_K - \rho_{Fl}) V_K g , \end{aligned} \quad (4)$$

¹Ernst Fritz H ppler (1897-1955)

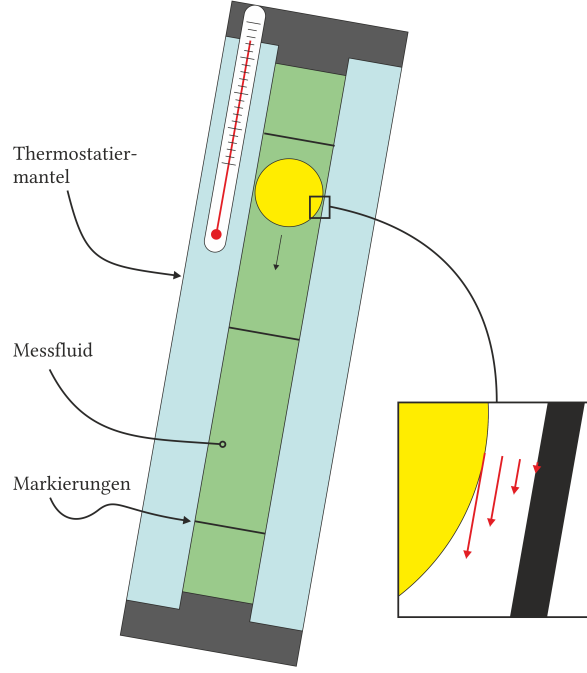


Fig. 2: Höppler's falling sphere viscometer

ρ_K and ρ_{Fl} are the densities of the sphere and the fluid, respectively, V_K is the volume of the sphere and g the gravitational acceleration. Combining eqs. (3) and (4), and using $v = s/t$ and $V_K = \frac{4}{3}\pi r^3$ one finds:

$$\eta = \frac{2r^2g}{9s} (\rho_K - \rho_{Fl}) t = K (\rho_K - \rho_{Fl}) t , \quad (5)$$

K is the sphere-constant and t is the time needed by the sphere to sink from the upper to the lower marker over the distance s . Since Stoke's equation (3) is not strictly valid for the falling sphere viscometer, a sphere-constant which has been experimentally calibrated is used in eq. (5). It is listed at the measuring station.

The temperature dependence of the viscosity of liquids follows an exponential law:

$$\frac{1}{\eta} = B \exp\left(-\frac{E_A}{RT}\right) \quad \text{or rearranged:} \quad \ln \eta = \frac{E_A}{RT} + \ln(B^{-1}) \quad (6)$$

where B is the so called pre-exponential factor, E_A is an activation energy, R is the universal gas constant and T is the absolute temperature.

2.3 Ostwald's capillary viscometer

Employing Ostwald's² capillary viscometer (fig. 3) the so-called kinematic viscosity ν is determined, which is connected to the dynamic viscosity η via the density ρ of the fluid:

²Wilhelm Ostwald (1853-1932)

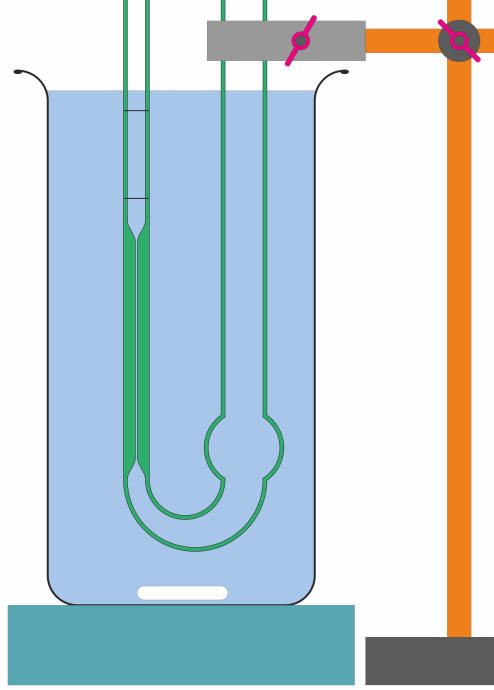


Fig. 3: Ostwald's capillary viscometer

$$\nu = \frac{\eta}{\rho} \quad (7)$$

The flow of a liquid in a capillary is well described by Hagen-Poiseuille's³ law:

$$\nu = \frac{\pi r^4 g \Delta h t}{8 l V} . \quad (8)$$

r is the inner radius of the capillary, g , again, the gravitational acceleration, Δh is the average height difference of the liquid column, t is the flow time, l is the capillary length and V the flow volume. This law is exactly valid only for capillaries of infinite length. The so-called Hagenbach-correction⁴ considers disturbing processes at the inlet and the outlet of the capillary:

$$\nu = \frac{\pi r^4 g \Delta h t}{8 l V} - \underbrace{\frac{m V}{8 \pi l t}}_{\text{Hagenbach-correction}} . \quad (9)$$

Here, the empirical and dimensionless factor m depends on the shape of the geometry of the inlet and especially of the outlet of the capillary.

³G. H. L. Hagen (1797-1884) und J. L. M. Poiseuille (1797-1869)

⁴E. Hagenbach-Bischoff (1833-1910)

2.4 Units and dimensions of the viscosity

	today's units	earlier used units
η	$\text{Pa} \cdot \text{s} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$	1 Poise = 0,1 Pa · s
ν	m^2s^{-1}	1 Stokes = $10^{-4} \text{m}^2\text{s}^{-1} = 1 \text{cm}^2\text{s}^{-1}$

3 Conducting the experiment

3.1 Falling sphere viscometer

First make sure that the thermostat contains sufficient water for temperature control (if not, refill with deionised water = „VE-Wasser“). Turn on the cooling water of the thermostat. Set the thermostat to the desired temperature (the upper edge of the small metal screw nut should point to this temperature). After about 10 min the temperature equilibrium between the water jacket and the internal tube is achieved.

Meanwhile the viscometer is aligned:

- one must be able to rotate it freely (attention: avoid pinching and twisting the water hoses),
- one must be able to read the thermometer inside the cooling jacket
- the viscometer has to be strictly aligned horizontally (that means: the air bubble inside the water spirit level has to be centered).
- Make sure there is no air bubble inside the internal tube of the viscometer (if there is call the assistant)

Measurement

Open the lock and rotate the entire water jacket with the internal tube by 180°. The sphere starts to fall down gently. Measure the time the sphere needs to fall from the upper to the lower marker. At each temperature (approx. 25, 30, 40 and 50 °C), five measurements need to be executed. The density of the sphere ρ_K and the constant K are given at the workplace. For the experiments at 40 and 50 °C the cooling cycle of the thermostat should be switched off.

3.2 Capillary viscometer

A big VE-water bath is used to control the temperature of the Ostwald capillary viscometer. The temperature of the water bath should be constant at $(20 \pm 1)^\circ\text{C}$. Fill it with warm water when temperature is lower than 20 °C or add ice when temperature is higher than 20 °C. To achieve temperature equilibration the water bath is stirred with a magnetic stirrer. The capillary has to be immersed in the water down to the upper marker. Temperature equalisation is achieved after a few minutes.

Messung

- Rinse the capillary 2-3 times with the liquid to be measured. For this purpose, about 3 ml liquid is filled in the viscometer via the wide tube (on the right in fig. 3) and then the liquid is sucked into the capillary using a flip style pipette filler (Peleus-Ball) or a rubber bulb connected to the smaller (left, fig. 3) tube.
- After rinsing, discard the liquid. Then refill again with 3 mL and suck the liquid upwards above the upper marker.
- Using a stop watch measure the time the liquid meniscus needs to drain off from the upper to the lower marker. Measure three times and control the temperature of the water bath.

Deionized water („VE-Wasser“) is used to calibrate the capillary viscometer. Then measure the viscosity of ethanol and a mixture of ethanol and water (1:1 in volume).

$$\nu_{\text{sample}} = \underbrace{\frac{\nu_{H_2O}(20^\circ\text{C})}{t_{H_2O}(20^\circ\text{C})}}_{\text{calibration factor}} \cdot t_{\text{sample}} - \underbrace{1,2 \cdot 10^{-7} \text{ m}^2 \cdot \frac{1}{t_{\text{sample}}}}_{\text{Hagenbach correction}} \quad (10)$$

The required data for $\eta_{H_2O}(20^\circ\text{C})$ and $\rho_{H_2O}(20^\circ\text{C})$ can be found in the appendix.

4 Data treatment

4.1 Falling sphere viscometer

1. Calculate the mean values and standard deviations of falling times t and temperatures T . Determine the respective dynamic viscosities η . Plot the dynamic viscosities versus temperatures including error bars. Compare your results to published values.
2. Plot $\ln \eta$ versus $1/T$ and determine the activation energy E_A .
3. Make a reasonable assumption for the radius r_{H_2O} of a water molecule and calculate the self diffusion coefficient D at 20°C using the Stokes-Einstein equation (11):

$$D = \frac{k_B T}{6\pi\eta r_{H_2O}} \quad (11)$$

4.2 Capillary viscometer

1. Determine the kinematic viscosities ν of the liquids using equation (10) and subsequently the dynamic viscosities η . All necessary data can be found in the appendix.
2. Try to interpret the surprising experimental results.

4.3 Additional questions

1. Arrange the following sequence of liquids with respect to increasing viscosity: water, acetone, glycerol, methanol. Give an interpretation of the sequence
2. Discuss the similarity of the relation of the temperature dependence of the viscosity of liquids and the Arrhenius equation for the temperature dependence of chemical reactions.

Hints

Please preserve in your report both the order and the nomenclature used in the script! When presenting your numeric results please check for the significant number of digits and do not forget the units! Accomplish an error calculation (start with the standard deviation of your primary data (measuring results) and calculate the error bars according to Gauß' error propagation.)

5 Appendix

The data can be taken, e.g., from *CRC-Handbook of Chemistry and Physics* (various editions are available)

$\eta_{H_2O}(20\text{ }^{\circ}\text{C})$	$1,000 \cdot 10^{-3} \text{ Pa}\cdot\text{s}$
$\rho_{H_2O}(20\text{ }^{\circ}\text{C})$	$998,3 \text{ kg} \cdot \text{m}^{-3}$
$\rho_{H_2O-EtOH}(20\text{ }^{\circ}\text{C})$	$929,6 \text{ kg} \cdot \text{m}^{-3}$
$\rho_{EtOH}(20\text{ }^{\circ}\text{C})$	$789,3 \text{ kg} \cdot \text{m}^{-3}$

6 What you should know

Transport laws, Newtonian and non-Newtonian fluids, experimental methods to measure viscosities, temperature and pressure dependence of the viscosities of gases and liquids.

7 Hints for the risk assessment

Analyse potential hazards that arise from the methods and substances used in this experiment.

Discuss:

- suggest rules of conduct (organizational and administrative rules)
- suggest technical safety precautions
- what personal protective equipment should be used?